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# Influence of mesh density on a finite element model under dynamic loading

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# **Influence of mesh density on a finite element model under dynamic loading**

**1. Introduction**

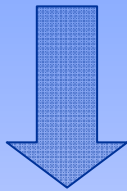
**2. Theoretical point of view**

**3. Simulations with different mesh density**

**4. Discussion and Conclusion**

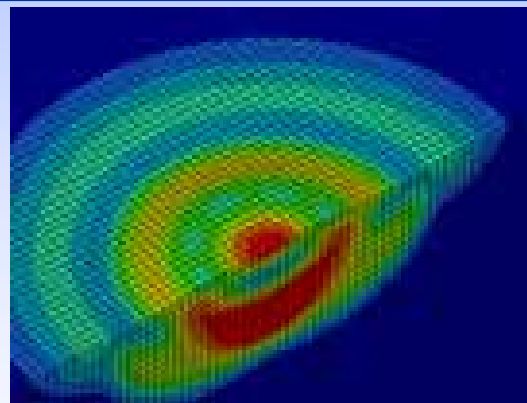
# Introduction

**Development of computer science**



**Use of numerical simulations (Finite Element Analysis) for physical problems**

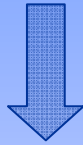
**Investigation of mechanical parameters at a local level**



**Civil Engineering  
Crash simulation  
Biomechanics**

# Introduction

**Dynamic simulations: Impact or wave propagation**



**Results of simulations are influenced by:**

**Mesh discretization**

**Material properties**

**Numerical dispersion**

**Spurious wave**

**Precision of the analysis**  
**Stability of the analysis**  
**Validity of the analysis**

# State of art

**Dynamic FE codes (Altair Radioss) with explicit integration methods such as Central Difference Method (CDM)**

Numerical Stability is assumed by the Courant Number (Hourglass Energy)

$$C = \frac{c \cdot \Delta t_{\max}}{\Delta x} \leq 1$$

the distance traveled by the fastest wave in the model ( $c \cdot \Delta t_{\max}$ ) should be smaller than the characteristic element size ( $\Delta x$ ) in the mesh

# State of art

**Courant's  
condition**

**Validity of the  
analysis**

**Even if the simulation is  
valid, there are  
numerical errors**

**Mesh size and wave direction lead to different  
results for a simulation**

*(Belytschko et al. 2000, Bazant et al. 1978)*

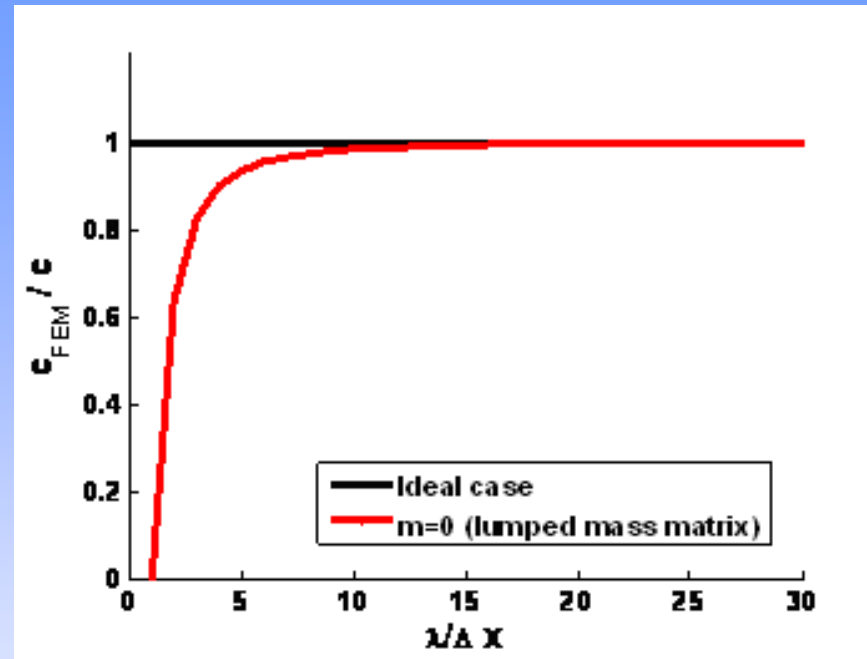
# Theoretical point of view

Ratio between theoretical and numerical wave velocity

$$\frac{c_{FEM}}{c} = \frac{2}{\Phi} \left[ \left( \sin \frac{\Phi}{2} \right)^{-2} - \frac{2}{3} m \right]^{-1/2}$$

$$\Phi = \frac{\omega \Delta x}{c} = \frac{2\pi \Delta x}{\lambda}$$

Bazant et al. 1978



For one dimensional simulation :  $\frac{c_{FEM}}{c} = 99\%$  for  $\frac{\lambda}{\Delta x} \sim 16$

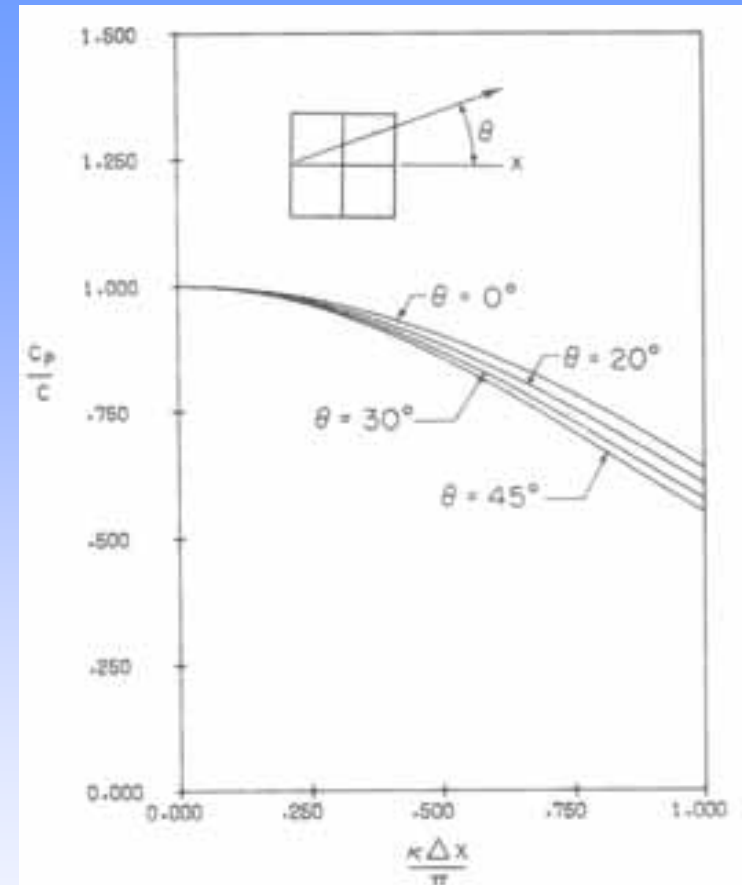
Numerical error due to the number of element per wavelength

# Theoretical point of view

**Spurious wave due to the direction  
of the propagation**

*(Belytschko et al. 1982)*

*Ratio between theoretical and  
numerical wave velocity for  
different incidence angle of  
the wave*



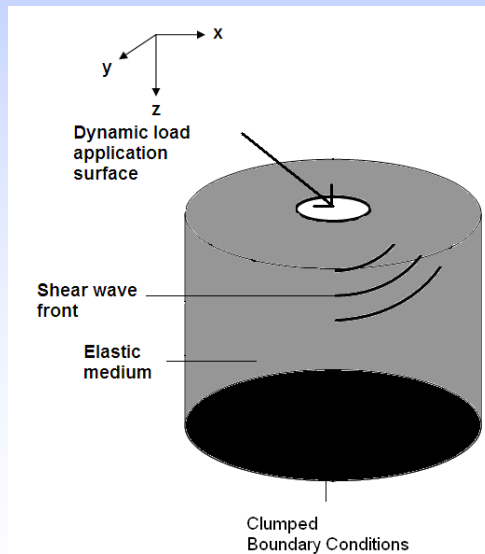
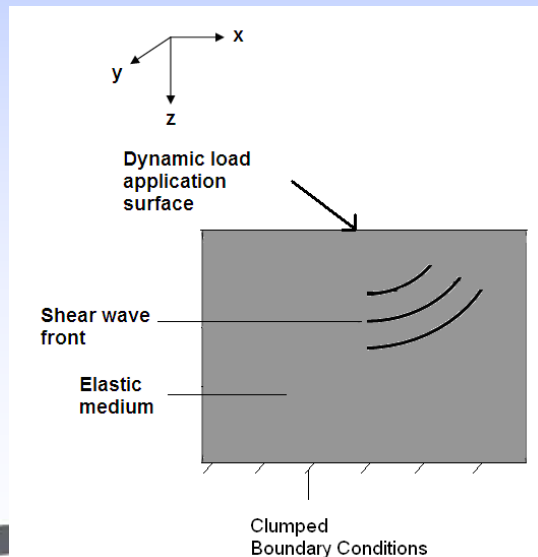
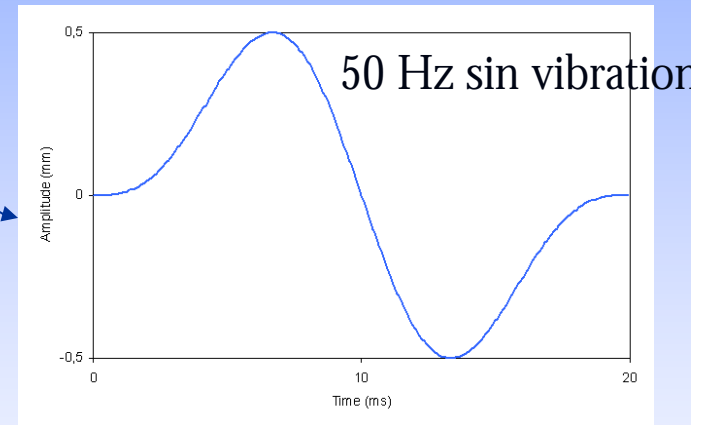
Numerical error due to the incident wave

# Simulations with different mesh density

Investigation of the mesh density in a FE model under dynamic load case

- Elasticity
- One single loading frequency

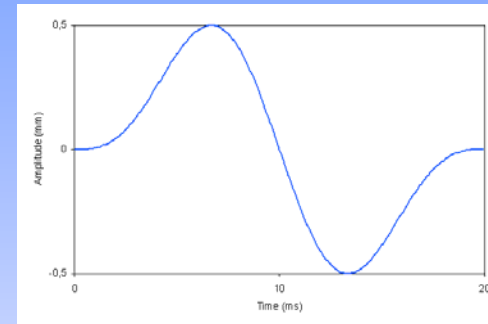
Young's Modulus (kPa)	4
Poisson's ratio	0.49
Density (kg/cm <sup>3</sup> )	1



# Simulations with different mesh density

Under the assumption of pure elasticity in the field of soft tissue

Young's Modulus (kPa)	4
Poisson's ratio	0.49
Density (kg/cm <sup>3</sup> )	1



$$V_s = \sqrt{\frac{E}{3\rho}}$$

shear wave velocity  
= 1,1 m/s

$$\lambda_s = \frac{V_s}{F_0}$$

Wavelength of shear wave  
= 23 mm

# Simulations with different mesh density

## Mesh density for 2D simulation

Models	Geometry of the mesh			number of element per wave length
	Plan dimension	Element size mm	Element type	
2D_QUAD_1	Width 90 mm Heigth 60 mm	0.25	4 nodes Quad elements	92
2D_QUAD_2		0.5		46
2D_QUAD_3		1		23
2D_QUAD_4		2		11
2D_QUAD_5		3		8
2D_QUAD_6		Bias		/

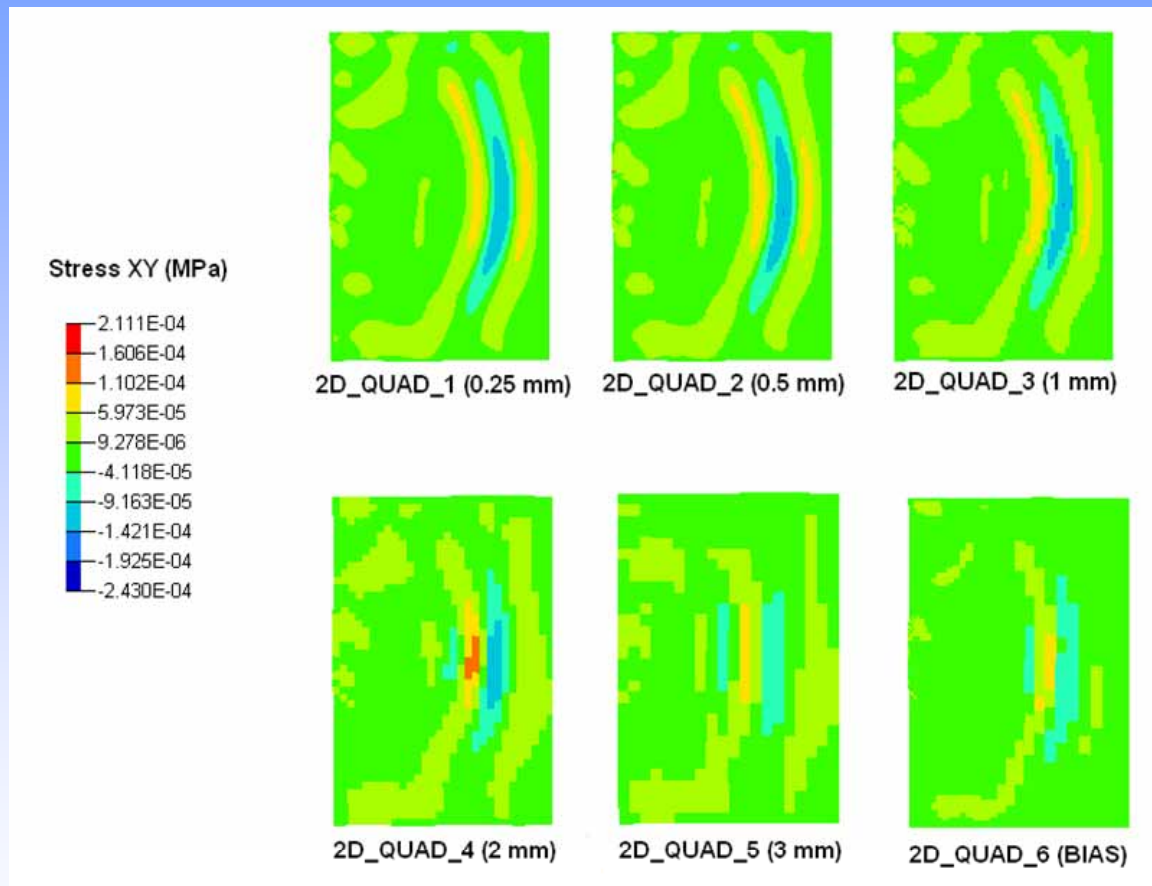
# Simulations with different mesh density

## Mesh density for 3D simulation

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# Simulations with different mesh density

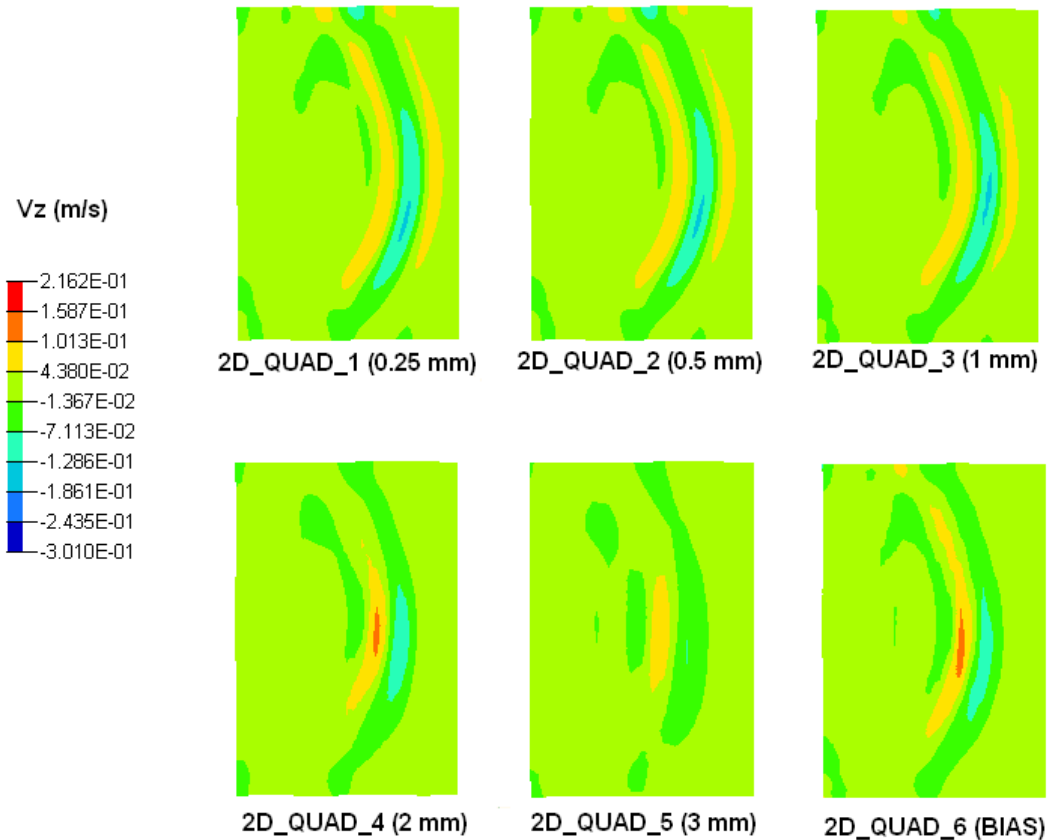
## Results for 2D simulations



Discrepancies in terms of stress distribution (55% difference)

# Simulations with different mesh density

## Results for 2D simulations

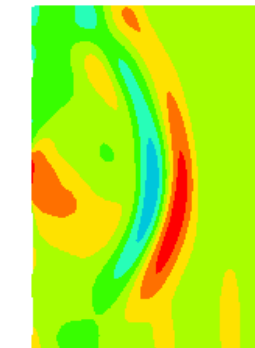
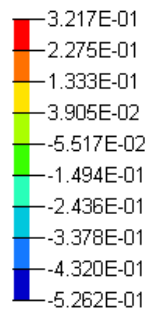


Discrepancies in terms of velocity  
( 50 % difference)

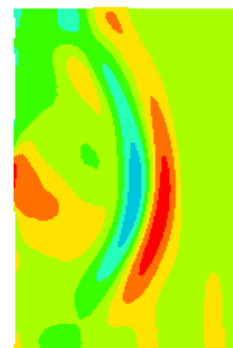
# Simulations with different mesh density

## Results for 2D simulations

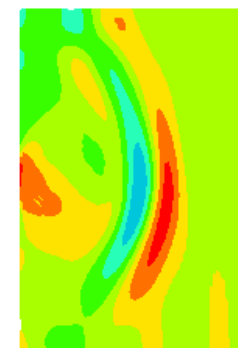
Displacement  
Z axis (mm)



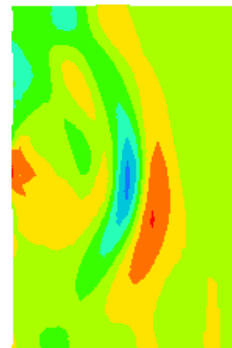
2D\_QUAD\_1 (0.25 mm)



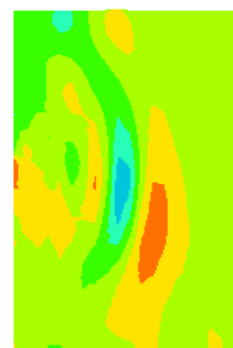
2D\_QUAD\_2 (0.5 mm)



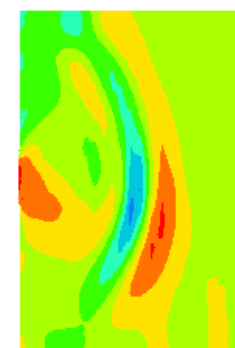
2D\_QUAD\_3 (1 mm)



2D\_QUAD\_4 (2 mm)



2D\_QUAD\_5 (3 mm)

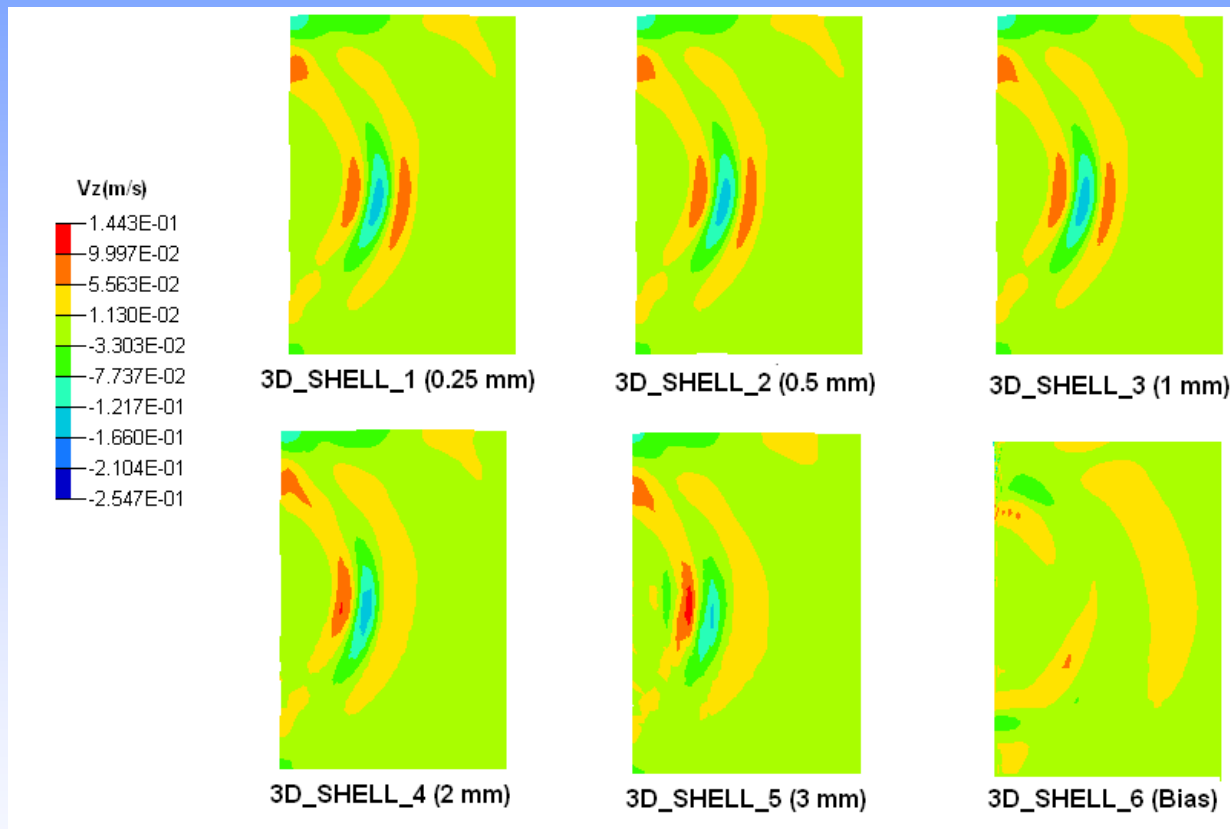


2D\_QUAD\_6 (BIAS)

Discrepancies in terms of displacement distribution (30% difference)

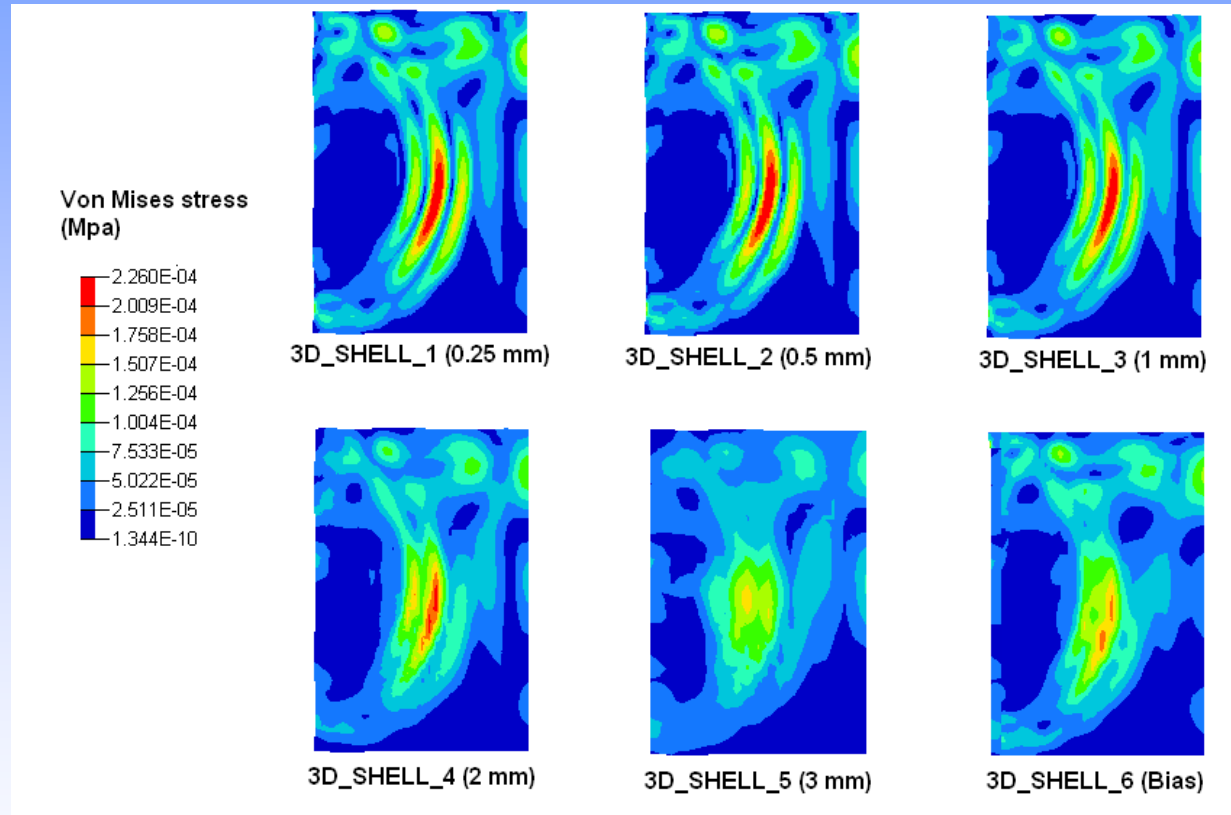
# Simulations with different mesh density

## Results for 3D simulations



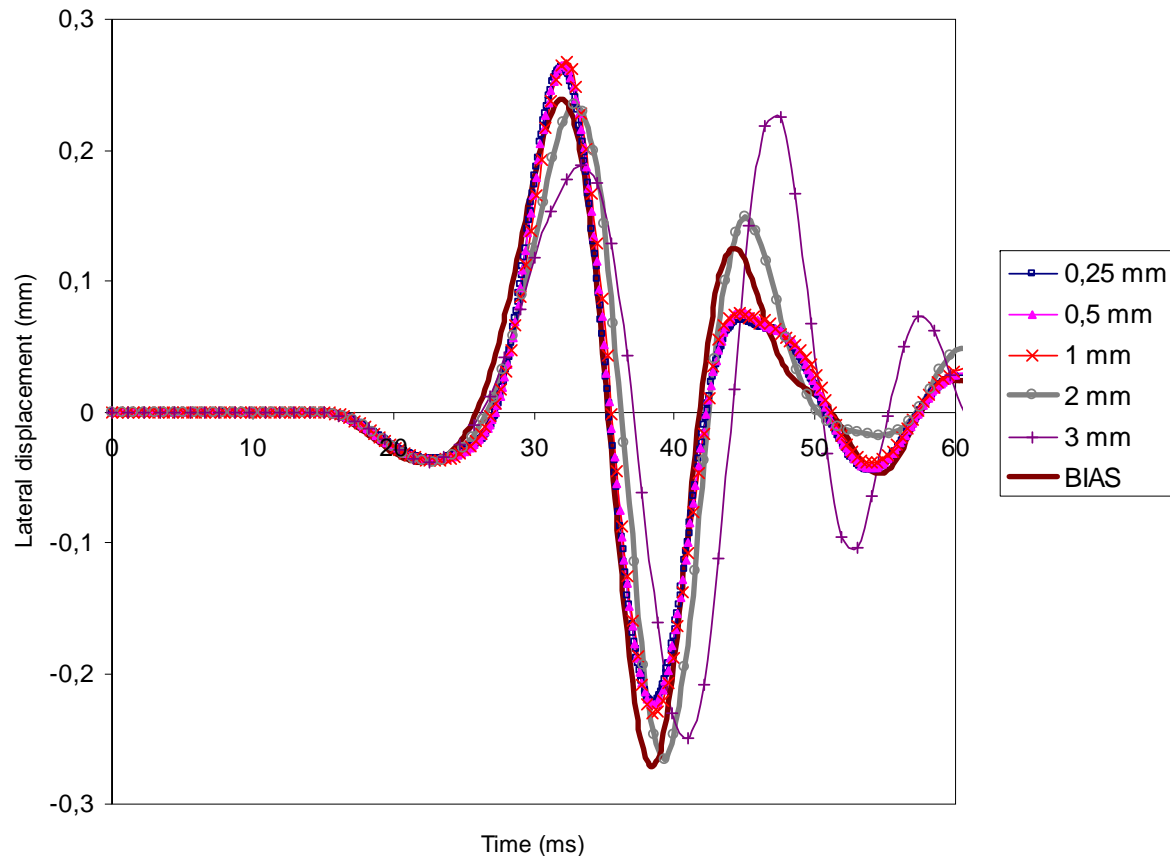
# Simulations with different mesh density

## Results for 3D simulations



# Simulations with different mesh density

## Results for 3D simulations



Equivalence of results  
for 92 (0,25 mm),  
46 (0,5 mm),  
24 (1 mm)  
elements per  
wavelength.

For others model  
discrepancies  
appear

# Simulations with different mesh density

Frequency dependence



Simulation performed with a 25 Hz sine vibration

Models	Geometry of the mesh			number of element per wave length
	Plan dimension	Element size mm	Element type	
3D_SHELL_1	Width 90 mm Heigth 60 mm	0.25	4 nodes Shell elements	184
3D_SHELL_2		0.5		92
3D_SHELL_3		1		46
3D_SHELL_4		2		23
3D_SHELL_5		3		16
3D_SHELL_6		Bias		/

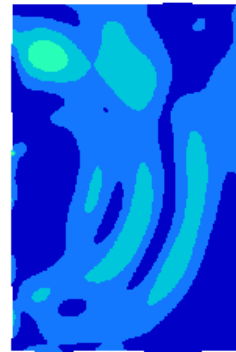
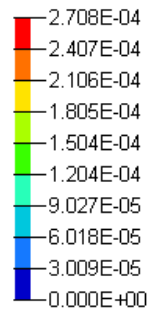
All the model have more than 16 elements per wavelength

# Simulations with different mesh density

Frequency dependence

Simulation performed with a 25 Hz sine vibration

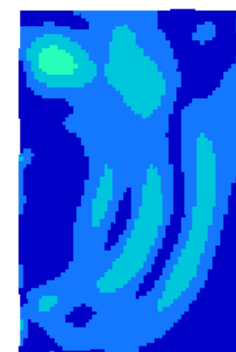
Von Mises Stress (MPa)



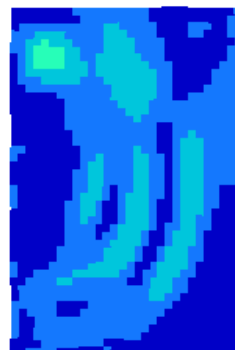
SHELL\_FEM1 (0.25 mm)



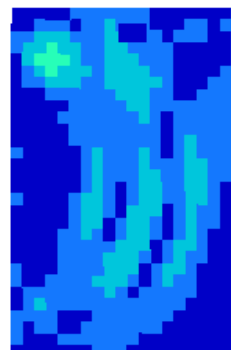
SHELL\_FEM2 (0.5 mm)



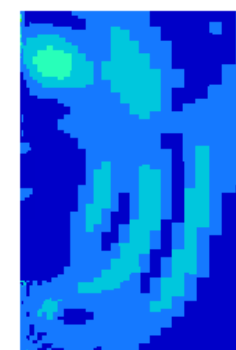
SHELL\_FEM3 (1 mm)



SHELL\_FEM4 (2 mm)



SHELL\_FEM5 (3 mm)



SHELL\_FEM6 (BIAS)

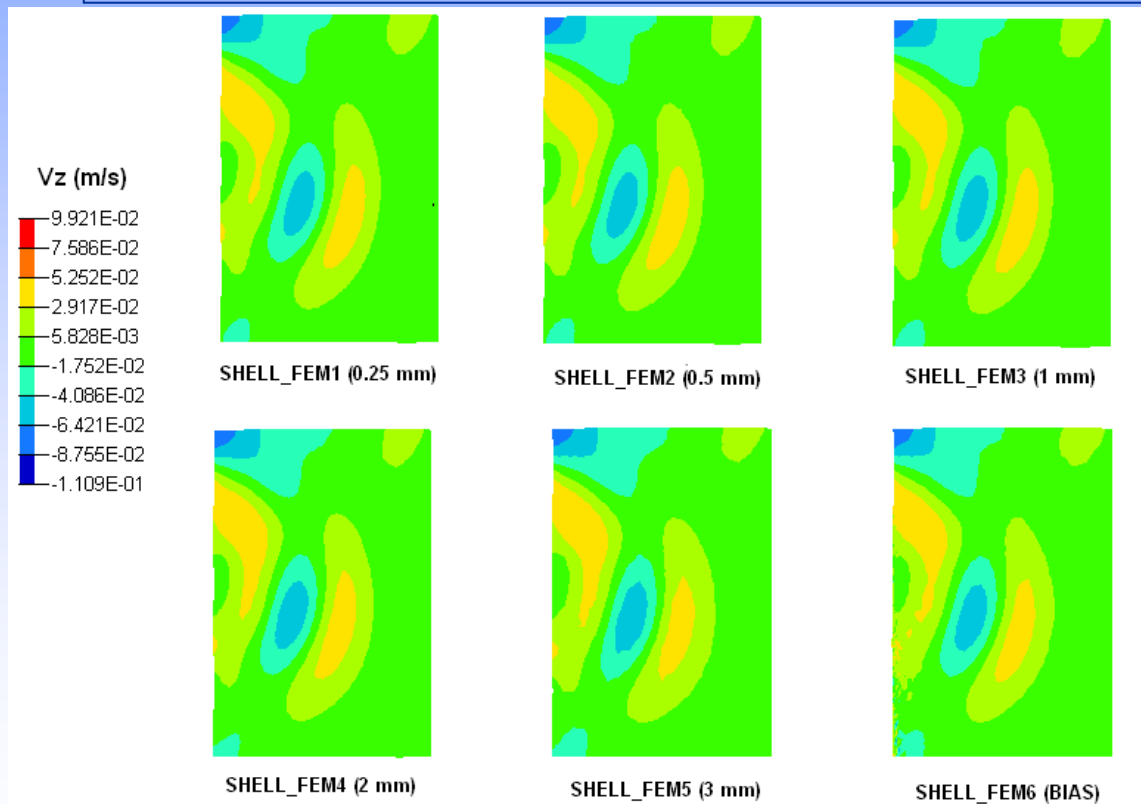
Equivalence of results for all models

# Simulations with different mesh density

Frequency dependence



Simulation performed with a 25 Hz sine vibration



Equivalence of results for all models

# Discussion and Conclusion

FE modelling

Discretization of the field

Mesh density ??

The study investigates the influence of mesh density with an explicit FE code (Radioss Altair©) : 15/16 element per wavelength for accurate results.

Element per wave length 15/16 elements (consistent with previous studies)

For an accurate discretization and a correct propagation of the wave.

More than 15/16 element per wavelength increase CPU time with no significant improvement of the results.

# Discussion and Conclusion

## Future work :

- Influence of element formulation (Number of integration points, anti hourglass formulation)
  - Investigation of multi frequency loading (crash simulation)
    - Investigation of the propagation angle of the wave
      - Simulation with more complex material laws

## Conclusion :

Great influence of the mesh on simple models.

For complex model (biomechanical models), a minimum number of element are necessary according to the loading case (frequency), with an important care on mesh homogeneity)